

Magnetization Oscillation of a Spinor Condensate Induced by Magnetic Field Gradient

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We study the spin mixing dynamics of ultracold spin-1 atoms in a weak non-uniform magnetic field with field gradient G , which can flip the spin from $+1$ to -1 so that the magnetization $m = \rho_+ - \rho_-$ is not any more a constant. The dynamics of $m_F = 0$ Zeeman component ρ_0 , as well as the system magnetization m , are illustrated for both ferromagnetic and polar interaction cases in the mean-field theory. We find that the dynamics of system magnetization can be tuned between the Josephson-like oscillation similar to the case of double well, and the interesting self-trapping regimes, i.e. the spin mixing dynamics sustains a spontaneous magnetization. Meanwhile the dynamics of ρ_0 may be sufficiently suppressed for initially imbalanced number distribution in the case of polar interaction. A "beat-frequency" oscillation of the magnetization emerges in the case of balanced initial distribution for polar interaction, which vanishes for ferromagnetic interaction.

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I. INTRODUCTION

Since the successful realization of ^{23}Na condensate in the optical trap [1], with the spin degrees of freedom liberated, the coherent spin-mixing dynamics inside a spin-1 BEC has been studied intensively [2–8]. The spin mixing interaction allows for exchanging atoms among spin components but conserving the total angular momentum. Two atoms in Zeeman state $|0\rangle$ can coherently scatter into the states $|1\rangle$ and $|-1\rangle$, and vice versa: $2|0\rangle \rightleftharpoons |1\rangle + |-1\rangle$. As one of the most active topics in quantum gases, such spin-exchange dynamics was first studied by Law *et. al.* [2] and has been observed in the way of population oscillations of the Zeeman states inside ^{87}Rb condensates [5], where atoms interact ferromagnetically. A temporal modulation of spin exchange interaction, which is tunable with optical Feshbach resonance, was recently proposed to localize the spin mixing dynamics in ^{87}Rb condensate [8].

The properties of a three-component ($F = 1$) spinor condensate are first studied by Ho [9] and Ohmi [10]. For a spin-1 system, atom-atom interaction takes the form $V(\mathbf{r}) = \delta(\mathbf{r})(c_0 + c_2\mathbf{F}_1 \cdot \mathbf{F}_2)$, where \mathbf{r} is the distance vector between two atoms, and c_0, c_2 denote spin-independent and spin-exchange interaction respectively. Many predictions are verified experimentally [11, 12], and the most fundamental property concerns the existence of two different phases determined by c_2 : the so-called polar ($c_2 > 0$) and ferromagnetic ($c_2 < 0$) states, corresponding to the $F = 1$ state of ^{23}Na and ^{87}Rb atomic condensates respectively. The fragmented condensate in a uniform magnetic field can be turned into a single condensate state by a field gradient [13, 14]. More exotic ground state phases both in the mean field level and the fully quantum many body theory have been extended to condensates with higher spins [15–17] and recently to spinor mixtures [18–20].

In this paper we study the dependence of the spin dynamics on a small magnetic field gradient, which practically provides a process to flip the spin between $+1$ and -1 states thus turns a fragmented condensate into a coherent one. We adopt the mean-field approximation, in which a spinor condensate is described by a multi-component vector field. It has provided surprisingly good descriptions for most properties of the spinor condensate as evidenced by the experimental verification of many predictions [4]. As the spin flipping term is considered in the condensate, the system magnetization exhibits obvious macroscopic oscillation similar to the Josephson oscillation of a scalar condensate in a double well [21] and we find that the dynamics of spin-0 component ρ_0 may be greatly suppressed in the case of polar interaction. This provides us an intriguing tool to manipulate the atomic population in spinor condensate.

II. THE EFFECTIVE HAMILTONIAN FOR THE SYSTEM

The many-body Hamiltonian of N spin-1 atoms of mass M in a uniform magnetic field reads

$$H = \sum_{k=1}^N \left(\frac{\mathbf{P}_k^2}{2M} + V_{\text{trap}} + \gamma \mathbf{B}_0 \cdot \mathbf{F}_k \right) + \sum_{k<l} V_{\text{int}}(\mathbf{r}_k - \mathbf{r}_l). \quad (1)$$

The atoms are loaded into an optical trap V_{trap} and V_{int} denotes the collisional interaction between atoms. \mathbf{P}_k and \mathbf{F}_k are the momentum operators and spin-1 operators of the k -th atom, respectively. We consider a field gradient G as has already been applied in the MIT experiment [11], and to be more specific we just replace \mathbf{B}_0 in (1) with

$$\mathbf{B}(\mathbf{r}) = B_0 \mathbf{r}_B = B_0 [\hat{\mathbf{z}} + G(x\hat{\mathbf{x}} - z\hat{\mathbf{z}})]. \quad (2)$$

We choose the local field direction \mathbf{r}_B as the spin quantization axis [13] by performing a unitary transformation $U = \prod_{k=1}^N e^{-\frac{i}{\hbar} \mathbf{n}(\mathbf{r}_k) \cdot \mathbf{F}_k}$ on the Hamiltonian (1) where

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$\mathbf{n} = \hat{\mathbf{z}} \times \mathbf{r}_B = Gx\hat{\mathbf{y}}$. The contact interaction V_{int} is spin conserving, hence it is invariant under the transformation. On the other hand, an additional Berry phase associated with the local change of basis emerges in the momentum term. Take the single particle hamiltonian as an example, $h = \frac{\mathbf{p}^2}{2M} + V_{trap} + \gamma\mathbf{B} \cdot \mathbf{F}$, and $U = e^{-\frac{i}{\hbar}GxF_y}$, we find that

$$\begin{aligned} U^\dagger \mathbf{p}^2 U &= (P_x - A)^2 + P_y^2 + P_z^2 \\ &= \mathbf{p}^2 - 2GF_y P_x + G^2 F_y^2 \end{aligned} \quad (3)$$

where the operator $A = GF_y$. The Zeeman energy term is transformed into [22]

$$\begin{aligned} U^\dagger \mathbf{B} \cdot \mathbf{F} U &= B_0 F_x [Gx \cos(Gx) - (1 - Gz) \sin(Gx)] \\ &+ B_0 F_z [Gx \sin(Gx) + (1 - Gz) \cos(Gx)]. \end{aligned} \quad (4)$$

For small field gradient G , we approximate the trigonometric functions up to order of $O(G^2)$ in accordance with Eq. (3), $\cos(Gx) \simeq 1 + G^2 x^2/2$, $\sin(Gx) \simeq Gx$, which leads us to

$$U^\dagger \mathbf{B} \cdot \mathbf{F} U = B_0[F_z + CG + DG^2 + O(G^3)] \quad (5)$$

where the operator $C = -zF_z$, $D = xzF_x + x^2F_z/2$.

The many-body Hamiltonian is finally transformed into

$$\begin{aligned} H_{eff} &= \sum_{k=1}^N \left\{ \frac{\mathbf{p}_k^2}{2M} - \frac{G}{M} F_y^k P_x^k + \frac{G^2}{2M} (F_y^k)^2 \right. \\ &+ V_{trap} + \gamma B_0 F_z^k + \gamma B_0 G C^k + \gamma B_0 G^2 D^k \} \\ &+ \sum_{k < l} V_{int}(\mathbf{r}_k - \mathbf{r}_l) \end{aligned} \quad (6)$$

The atomic interaction takes the form [9, 10]

$$V_{int}(\mathbf{r}) = (c_0 + c_2 \mathbf{F}_1 \cdot \mathbf{F}_2) \delta(\mathbf{r})$$

and $c_0 = 4\pi\hbar^2(a_0 + 2a_2)/3M$, $c_2 = 4\pi\hbar^2(a_2 - a_0)/3M$. The representation of the Hamiltonian in the second quantized form reads

$$\begin{aligned} \hat{H}_{eff} &= \int d\mathbf{r} \left\{ \hat{\Psi}_i^\dagger \left[\frac{\mathbf{p}^2}{2M} \delta_{ij} - \frac{G}{M} P_x (F_y)_{ij} + \frac{G^2}{2M} (F_y^2)_{ij} \right. \right. \\ &+ V_{trap} + p_0 (F_z)_{ij} + p_0 G C_{ij} + p_0 G^2 D_{ij} \left. \right] \hat{\Psi}_j \\ &+ \frac{c_0}{2} \hat{\Psi}_i^\dagger \hat{\Psi}_j^\dagger \hat{\Psi}_j \hat{\Psi}_i \\ &+ \frac{c_2}{2} \hat{\Psi}_i^\dagger \hat{\Psi}_{i'}^\dagger \mathbf{F}_{ij} \cdot \mathbf{F}_{i'j'} \hat{\Psi}_{j'} \hat{\Psi}_j \left. \right\} \end{aligned} \quad (7)$$

where repeated indices are to be summed over and $\hat{\Psi}(\mathbf{r})(\hat{\Psi}^\dagger(\mathbf{r}))$ is the field operator that annihilates (creates) an atom in the i -th hyperfine states with $i = 1, 0, -1$ at location \mathbf{r} , and $p_0 = \gamma B_0$ with γ the gyromagnetic ratio of the bosonic atoms.

The dynamics of the condensate components reveal a rich coupling between the spin and spatial degrees of freedom resulting in a variety of interesting phenomena, including spin mixing, spin domain formation and spin textures [4]. The internal and external dynamics are both very sensitive to the external magnetic fields and field gradients and they can be decoupled under certain conditions, in particular, when the available spin dependent interaction is insufficient to create spatial spin structures in the condensates. This occurs when the spin healing length is larger than the size of the condensate, which allows us to focus on the coherent spin mixing oscillation of the spin populations. Practically we choose a proper field gradient G to induce an energy in the same magnitude of spin interaction term c_2 . The gradient will flip the spin of atoms in the condensate but keep the three components still miscible and free of spin texture ($G < 2\text{cm}^{-1}$ as in the MIT experiment [11]). As a result we can still safely adopt the single spatial mode approximation (SMA) in the following [2–4, 7, 23].

We take

$$\hat{\Psi}_i^\dagger = \sqrt{N} \phi(\mathbf{r}) \hat{a}_i^\dagger \quad (8)$$

with $\phi(\mathbf{r})$ is defined by the Gross-Pitaevskii equation through the spin-independent part \hat{H}_0

$$\left(-\frac{\hbar^2 \nabla^2}{2M} + V_{trap} + c_0 N |\phi|^2 \right) \phi = \hat{H}_0 \phi = \mu \phi \quad (9)$$

where N is the total number of the atoms and μ is the mean field energy or the chemical potential. The $\hat{a}_i^\dagger(\hat{a}_i)$ is the spin component operator that annihilates (creates) an atom with spin i ($i = 1, 0, -1$). Substitute the ansatz (8) into the Hamiltonian (7) and neglect the spin-independent part, our model then reads

$$H_{eff} = -\epsilon(\hat{a}_1^\dagger \hat{a}_{-1} + \hat{a}_{-1}^\dagger \hat{a}_1) + \epsilon \hat{a}_0^\dagger \hat{a}_0 + c_2 \hat{\mathbf{F}}^2 - p \hat{F}_z \quad (10)$$

with

$$\begin{aligned} \hat{\mathbf{F}}^2 &= \hat{F}_z^2 + (\hat{F}_+ \hat{F}_- + \hat{F}_- \hat{F}_+)/2 \\ \hat{F}_\pm &= \sqrt{2}(\hat{a}_{\pm 1}^\dagger \hat{a}_0 + \hat{a}_0^\dagger \hat{a}_{\mp 1}) \\ \hat{F}_z &= \hat{a}_1^\dagger \hat{a}_1 - \hat{a}_{-1}^\dagger \hat{a}_{-1}. \end{aligned} \quad (11)$$

The parameter $\epsilon = G^2/4M$ characterizes the spin-flipping process induced by the field gradient, and the spin interaction parameter is scaled as $c'_2 = (c_2/2) \int d\mathbf{r} |\Phi(r)|^4$ and we keep the original notation c_2 for ease of representation. The term C_{ij} vanishes due to the fact that in ground state ϕ is a symmetric function ($\int d\mathbf{r} \phi^* x \phi = 0$ and $\int d\mathbf{r} \phi^* z \phi = 0$, etc.), so does the term $P_x (F_y)_{ij}$ because

$$\int d\mathbf{r} \phi^* P_x \phi = \frac{M}{i\hbar} \int d\mathbf{r} \phi^* [x, H_0] \phi = 0.$$

The term D_{ij} amounts to a shift of the linear Zeeman energy $p = p_0 + \tilde{p}$ with $\tilde{p} = p_0 G^2 \int d\mathbf{r} \phi^* x^2 \phi$.

The ϵ term in Hamiltonian (10) denotes the process that flips the spin from 1 to -1 or vice versa, and it plays the same role as the hopping term of scalar condensates in a double well. We want to emphasize that this term in the form of $\hat{a}_i^\dagger (F_y^2)_{ij} \hat{a}_j$ induces the oscillation of the z -component magnetization m and goes against the quadratic Zeeman effect term $\hat{a}_i^\dagger (F_z^2)_{ij} \hat{a}_j$ in Refs. [4, 7], which instead adheres to the constancy of the magnetization m and keep the system in the polar phase [11]. In a uniform field, the z -component magnetization m is a constant, and the system can be described by two canonical conjugate variables: the population on spin-0 component ρ_0 and the relative phase $\theta = \theta_1 + \theta_{-1} - 2\theta_0$. The system can be finally reduced to a nonrigid pendulum model [4]. Our model system conserves only the total number of atom and another relative phase $\theta' = \theta_1 - \theta_{-1}$ arises. We will mainly study the dynamics of the population on spin-0 component $\rho_0(t) = N_0(t)/N$, and the z -component magnetization $m(t) = (N_1(t) - N_{-1}(t))/N$. The populations on \pm components are related to ρ_0 and m through $\rho_1 + \rho_{-1} + \rho_0 = 1$ and $m = \rho_1 - \rho_{-1}$.

III. THE SEMICLASSICAL MODEL

Instead of considering a Hilbert space of the Fock states $|N_1, N_0, N_{-1}\rangle$, where the population dynamic can be describe as

$$\rho_0 = \langle \psi_i | e^{i\hat{H}t/\hbar} \hat{a}_0^\dagger \hat{a}_0 e^{-i\hat{H}t/\hbar} | \psi_i \rangle / N$$

with $|\psi_i\rangle$ an initial state which can be taken as one of the basis in the Hilbert space, here we consider the condensate to be in a coherent state, associated with a macroscopic wave function with both magnitude and phase. In the study of spin mixing dynamics [3, 7, 8], it is customary to replace the operator $\hat{a}_i^\dagger (\hat{a}_i)$ by c numbers $a_i^* (a_i)$, and the coherent state is analogous to a classical field of complex amplitude with a definite phase in each spin component

$$|\Phi\rangle = |a_1, a_0, a_{-1}\rangle = \left| \sqrt{N_1} e^{i\theta_1}, \sqrt{N_0} e^{i\theta_0}, \sqrt{N_{-1}} e^{i\theta_{-1}} \right\rangle$$

Semiclassical equations of motion can be derived from Hamiltonian (10) as

$$\begin{aligned} i\hbar \dot{a}_1 &= 2c_2(\tilde{F}_z a_1 + \tilde{F}_- a_0/\sqrt{2}) - \epsilon a_{-1} - p a_1 \\ i\hbar \dot{a}_0 &= \sqrt{2}c_2(\tilde{F}_+ a_1 + \tilde{F}_- a_{-1}) + \epsilon a_0 \\ i\hbar \dot{a}_{-1} &= 2c_2(-\tilde{F}_z a_{-1} + \tilde{F}_+ a_0/\sqrt{2}) - \epsilon a_{+1} + p a_{-1} \end{aligned} \quad (12)$$

where the quantities $\tilde{F}_\pm, \tilde{F}_z$ are c number counterparts of the operators \hat{F}_\pm and \hat{F}_z respectively. The neglected spin-independent part of the Hamiltonian (7) give each of the three equations in Eq. (12) a constant energy shift μ that can be trivially eliminated by changing a_i to $a_i e^{-i\mu t/\hbar}$. For simplicity we rescale the time as $t \rightarrow |c_2| t/\hbar$. The three coupled Gross-Pitaevskii equations

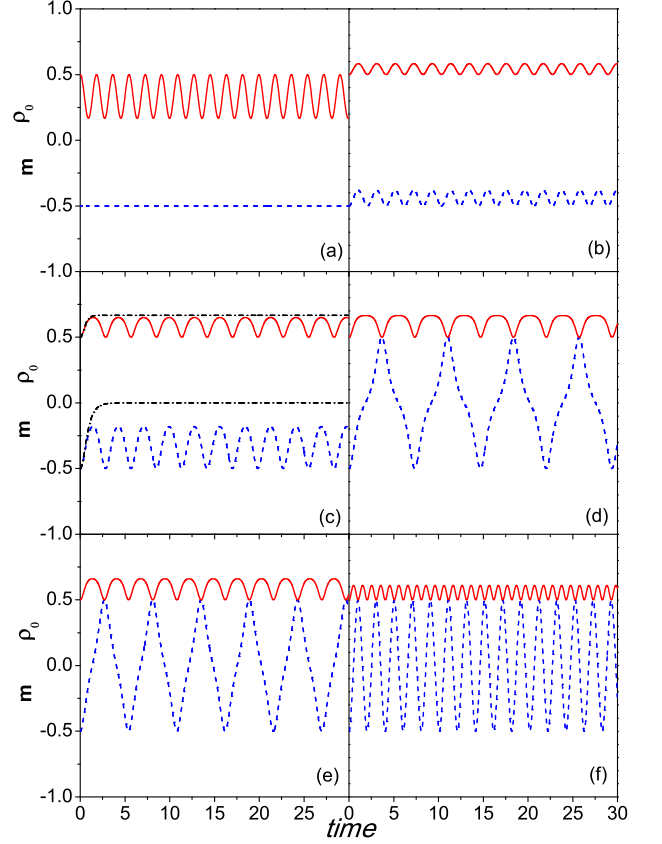


FIG. 1: (Color online) The dependence of the dynamics of $\rho_0(t)$ (red solid) and $m(t)$ (blue dashed) on the parameters of ϵ at fixed values of $p = 0$, $c_2 = 1$ (in units of $|c_2|$), and $\epsilon = 0$ (a), 1.45 (b), 1.49, 1.50 (c), 1.51 (d), 1.55 (e) and 2.25 (f). Fig.1(c) shows the critical transition parameters of $\rho_0(t), m(t)$ with $\epsilon = 1.50$ (black dashed dot line). Time is in units of $|c_2| t/\hbar$.

(12) for the interacting condensate amplitudes $a_i^* (a_i)$ describe the dynamics in terms of the inter-component phase difference and population imbalance.

IV. RESULTS AND DISCUSSION

First, we consider an initial distribution with imbalance between $|+1\rangle$ and $|-1\rangle$ components, i.e. $|\Phi(0)\rangle = |0, \sqrt{N/2} e^{i\theta_0}, \sqrt{N/2} e^{i\theta_{-1}}\rangle$ and illustrate the dynamics of population $\rho_0(t)$ and magnetization $m(t)$ for both $c_2 > 0$ and $c_2 < 0$. In order to give prominence to the effect of the ϵ term, we consider $p = 0$ first, and set the phases $\theta_0 = \theta_{-1} = 0$. This initial imbalance $N_1(0) - N_{-1}(0) = -N/2$ provides a "Junction voltage" and the magnetization oscillation was induced by ϵ . In Fig. 1, we show the solutions of equations (12) for $c_2 > 0$ and illustrative parameters $\epsilon = 0, 1.45, 1.49, 1.50, 1.51, 1.55$ and 2.25 , in the unit of $|c_2|$, respectively. We find that at the very

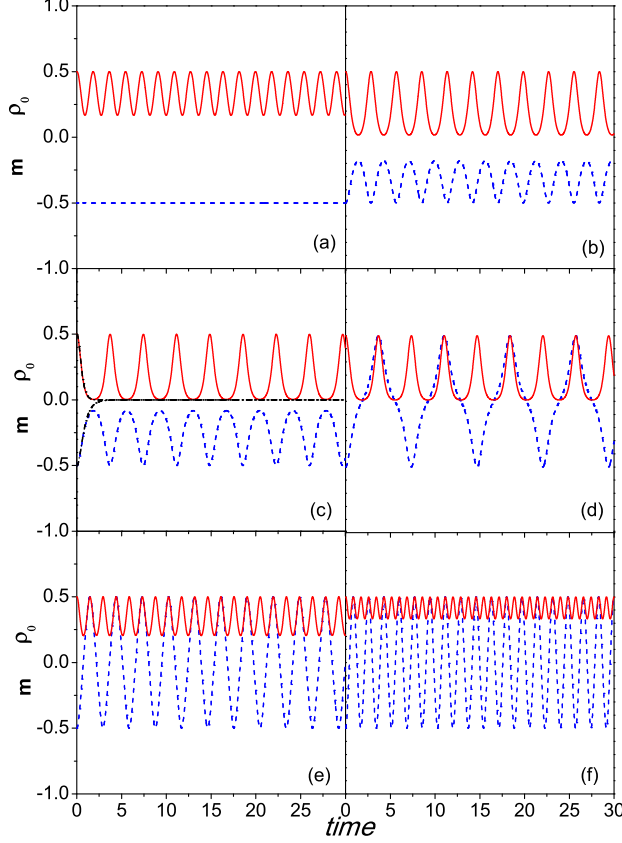


FIG. 2: (Color online) The dependence of the dynamics of $\rho_0(t)$ (red solid) and $m(t)$ (blue dashed) on the parameters of ϵ at fixed values of $p = 0$, $c_2 = -1$ (in units of $|c_2|$), and $\epsilon = 0$ (a), 0.45 (b), 0.49, 0.50 (c), 0.51 (d), 0.85 (e), and 1.50 (f). Fig.2 (c) shows the critical transition parameters of $m(t)$ with $\epsilon = 0.50$ (black dashed dot line). Time is in units of $|c_2|t/\hbar$.

beginning when ϵ is small the magnetization oscillates with small amplitude around an equilibrium above the initial value of $m(0) = -0.5$ as in Fig.(1b), which is analogous to the "macroscopic quantum self-trapping" effect in double well system [21]. Meanwhile the dynamics of $\rho_0(t)$ experiences a crossover from sinusoidal to non-sinusoidal oscillation, with the population $\rho_0(t)$ averaged over time changing from less than the initial value $\rho_0(0) = 0.5$ to larger than it. As ϵ increases, there is a critical transition for $\epsilon = 1.50$, black dashed line in Fig.(1c), then the oscillation extends to the range between -0.5 and 0.5 . Accompanied by the arising of the "Josephson tunneling" [21] of the magnetization, the dynamics of ρ_0 has been sufficiently suppressed in Fig.(1f). The coherent scattering of the internal Zeeman components $2|0\rangle \rightleftharpoons |1\rangle + |-1\rangle$ was suppressed by the ϵ term with the process $|1\rangle \rightleftharpoons |-1\rangle$. The critical behavior depends on ϵ , as can be easily found from the energy conservation and the extreme point for the minimization of the energy. Considering an arbitrary wave function

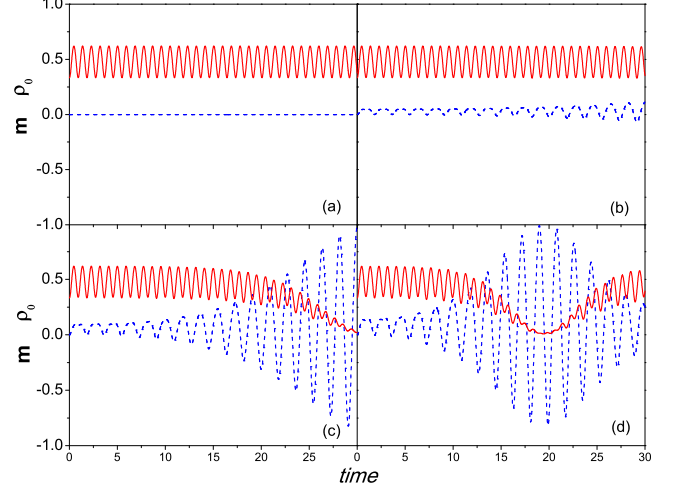


FIG. 3: (Color online) The dependence of the dynamics of $\rho_0(t)$ (red solid) and $m(t)$ (blue dashed) on the parameters of p at fixed values of $\epsilon = 1.55$, $c_2 = 1$ (in units of $|c_2|$), and $p = 0$ (a), 0.2 (b), 0.4 (c), 0.6 (d). Time is in units of $|c_2|t/\hbar$.

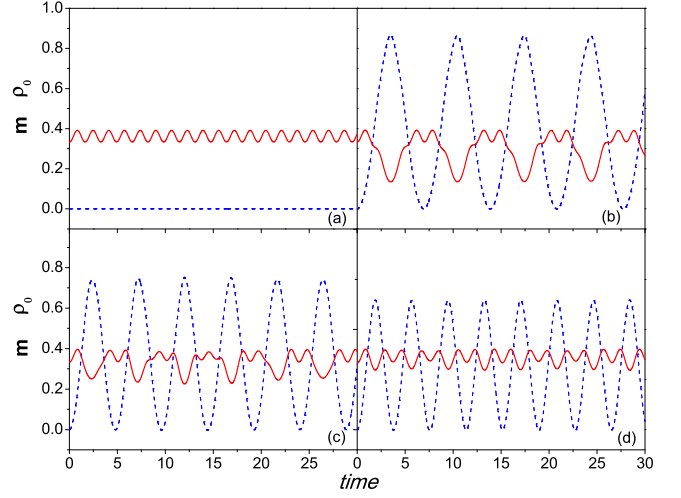


FIG. 4: (Color online) The dependence of the dynamics of $\rho_0(t)$ (red solid) and $m(t)$ (blue dashed) on the parameters of p at fixed values of $\epsilon = 0.55$, $c_2 = -1$ (in units of $|c_2|$), and $p = 0$ (a), 0.2 (b), 0.4 (c), 0.6 (d). Time is in units of $|c_2|t/\hbar$.

$|\Phi\rangle = |xe^{i\theta_1}, ye^{i\theta_0}, ze^{i\theta_{-1}}\rangle$, the relative average energy of the system when $p = 0$ can be described as

$$E = c_2[2y^2(x^2 + z^2 + 2xz \cos \theta) + (x^2 - z^2)^2] - 2\epsilon xz \cos \theta' + \epsilon y^2 \quad (13)$$

with $\theta = \theta_+ + \theta_{-1} - 2\theta_0$, and $\theta' = \theta_+ - \theta_{-1}$. For $c_2 > 0$, the critical point favors that $y = \sqrt{2/3}$, $x = z = \sqrt{1/6}$, $\theta = \pi$, $\theta' = \pi$. According to the energy conservation condition $E_c = E_{initial}$, we can get the critical value $\epsilon_c/c_2 = 1.50$.

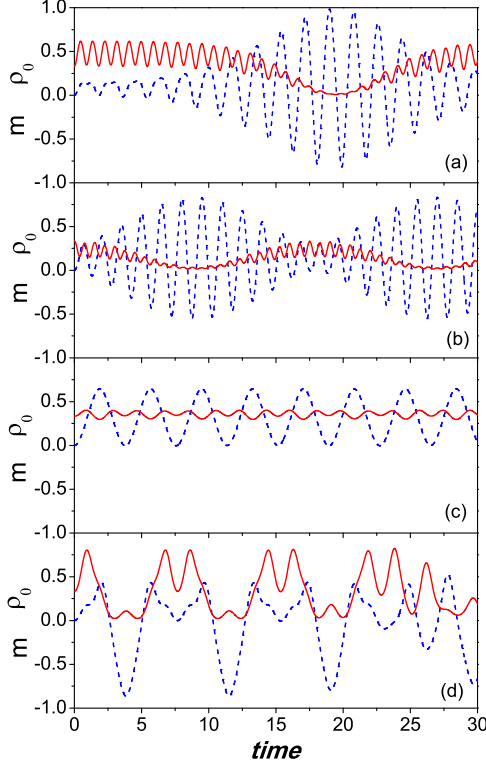


FIG. 5: (Color online) The dependence of the dynamics of $\rho_0(t)$ (red solid) and $m(t)$ (blue dashed) on the phases of the initial condition $|\Phi(0)\rangle = |\sqrt{N/3}e^{i\theta_1}, \sqrt{N/3}e^{i\theta_0}, \sqrt{N/3}e^{i\theta_{-1}}\rangle$ with parameters of $c_2 = 1$, $\epsilon = 0.55$, $p = 0.6$ and $\theta_1 = \theta_0 = \theta_{-1} = 0$ for (a), $c_2 = 1$, $\epsilon = 0.55$, $p = 0.6$ and $\theta_1 = \theta_{-1} = \pi/2$, $\theta_0 = \pi$ for (b), $c_2 = -1$, $\epsilon = 0.55$, $p = 0.6$ and $\theta_1 = \theta_{-1} = \theta_0 = 0$ for (c), $c_2 = -1$, $\epsilon = 0.55$, $p = 0.6$ and $\theta_1 = \theta_{-1} = \pi/2$, $\theta_0 = \pi$ for (d). Time is in units of $|c_2|t/\hbar$.

For $c_2 < 0$, we illustrate the dynamics for different parameters in Fig.(2) with $\epsilon = 0, 0.45, 0.49, 0.50, 0.51, 0.85$ and 1.50 , in the unit of $|c_2|$ respectively. We find that the oscillation of $m(t)$ here is almost the same as in the polar interaction case $c_2 > 0$. On the other hand, $\rho_0(t)$ is not suppressed at the beginning (Fig.(2b)), instead, it was enhanced. When $\epsilon > 0.5$, and the oscillation of ρ_0 is still active until ϵ reaches 1 or even large value. This ferromagnetic feature is quite different from the $c_2 > 0$ case. The latter case shows some "repulsive" effect between the Zeeman components. The critical value $\epsilon_c/(-c_2) = 0.5$ is derived analytically through the equation (13) with the extreme point $y = 0, x = z = \sqrt{1/2}$, $\theta = \theta' = 0$.

Next, we consider a balanced initial distribution with $|\Phi(0)\rangle = |\sqrt{N/3}e^{i\theta_1}, \sqrt{N/3}e^{i\theta_0}, \sqrt{N/3}e^{i\theta_{-1}}\rangle$ and $\theta_1 = \theta_{-1} = \theta_0 = 0$, where the "Junction voltage" between the $|1\rangle$ and $|-1\rangle$ components vanishes. We consider the

effect of the parameter p , while we choose a fixed value of $\epsilon = 1.55$ (in the unit of $|c_2|$) for $c_2 > 0$ and $\epsilon = 0.55$ for $c_2 < 0$ as shown in Fig.(3) and Fig.(4).

We find that the p term acts as a switch for the dynamics of the magnetization $m(t)$, and the cases for $c_2 > 0$ and $c_2 < 0$ are quite different. For $c_2 > 0$ in Fig.(3), the fast oscillation of $m(t)$ is modulated by a beat frequency, and, when the amplitude of the envelope function reaches its maximum the population of ρ_0 is completely suppressed to zero. For the $c_2 < 0$ case, the oscillation of $m(t)$ is also induced by p , but the magnetization is always positive. The dynamics of ρ_0 shows completely anharmonic behavior with the amplitude first enhanced then reduced as p increases, and no beat frequency modulation of the $m(t)$ occurs.

However, specific features of quantum nature cannot be addressed satisfactorily within a mean-field treatment. In an early experiment [5] on an $F = 1$ ^{87}Rb condensate, atoms all prepared initially in the state $|0, N, 0\rangle$ are observed to exhibit a damped oscillation accompanied by large fluctuations during the spin-mixing evolution. The dynamics of a polar initial state $|0, N, 0\rangle$ is trivial, i.e. the population ρ_0 remains a constant, within a mean-field treatment, but many-body quantum dynamics shows interesting damped oscillation [2]. The presence of a field gradient will enlarge the Hilbert space in quantum treatment due to the failure of the conservation of magnetization m and the related calculation on this feature will be published elsewhere.

Finally, let's consider the effect of the phase difference between the three components. Fig.(5) shows the dynamics in the presence of an initial phase difference for both $c_2 > 0$ and $c_2 < 0$ cases. We find that the influence of the phase difference for both cases are obvious. The beat frequency oscillation of the magnetization remains in the case of $c_2 > 0$ with a shift of the envelop center, but for the $c_2 < 0$ the phase difference changes the amplitude of $m(t)$ which extends down to the negative part of the axis and the dynamics of ρ_0 is also altered drastically.

V. CONCLUSION

The dynamics of spin-1 BEC in a nonuniform magnetic field is studied with the emergence of an additional spin-flipping term induced by the field gradient, which has an effect to reverse the spin from $+1$ to -1 and vice versa. Due to this spin flipping process the system magnetization $m(t)$ is not a constant any more, instead, it shows characteristic oscillation identical to that of a scalar BEC in double well. Meanwhile, the dynamics of ρ_0 was greatly altered. We present the dynamics of $\rho_0(t)$ and $m(t)$ under different initial conditions and the effect of phase difference is also shown. We find that the results for the polar ($c_2 > 0$) and ferromagnetic ($c_2 < 0$) cases are quite different. The small magnetic field gradient is chosen properly to give rise to the flipping of the spin between $+1$ and -1 but still keep the three components

miscible. These results highlight the possibility to manipulate the coherent dynamics of the spinor condensate with a field gradient, which is accessible to the current experimental techniques.

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- [1] D. M. Stamper-Kurn, M. R. Andrews, A. P. Chikkatur, S. Inouye, H.-J. Miesner, J. Stenger, and W. Ketterle, *Phys. Rev. Lett.* **80**, 2027 (1998).
 - [2] C. K. Law, H. Pu, and N. P. Bigelow, *Phys. Rev. Lett.* **81**, 5257 (1998).
 - [3] H. Pu, C. K. Law, S. Raghavan, J. H. Eberly, and N. P. Bigelow, *Phys. Rev. A* **60**, 1463 (1999); H. Pu, S. Raghavan, and N. P. Bigelow, *ibid.* **61**, 023602 (2000).
 - [4] M.-S. Chang, Q. S. Qin, W. X. Zhang, L. You, and M. S. Chapman, *Nat. Phys.* **1**, 111 (2005); W.X. Zhang, D. L. Zhou, M.-S. Chang, M.S. Chapman, and L. You, *Phys. Rev. A* **72**, 013602 (2005).
 - [5] M.-S. Chang, C. D. Hamley, M. D. Barrett, J. A. Sauer, K.M. Fortier, W. Zhang, L. You, and M. S. Chapman, *Phys. Rev. Lett.* **92**, 140403 (2004).
 - [6] R. Cheng, J.-Q. Liang and Y. Zhang, *J. Phys. B: At. Mol. Opt. Phys.* **38**, 2569 (2005).
 - [7] L. Chang, Q. Zhai, R. Lu, and L. You, *Phys. Rev. Lett.* **99**, 080402 (2007); Q. Zhai, L. Chang, R. Lu, and L. You, *Phys. Rev. A* **79**, 043608 (2009).
 - [8] W.X. Zhang, B. Sun, M. S. Chapman, and L. You, *Phys. Rev. A* **81**, 033602 (2010).
 - [9] T.-L. Ho, *Phys. Rev. Lett.* **81**, 742 (1998).
 - [10] T. Ohmi and K. Machida, *J. Phys. Soc. Jpn.* **67**, 1822 (1998).
 - [11] J. Stenger, S. Inouye, D. M. Stamper-Kurn, H.-J. Miesner, A. P. Chikkatur and W. Ketterle, *Nature (London)* **396**, 345 (1998).
 - [12] L. E. Sadler, J. M. Higbie, S. R. Leslie, M. Vengalattore, and D. M. Stamper-Kurn, *Nature (London)* **443**, 312 (2006).
 - [13] T.-L. Ho and S.-K. Yip, *Phys. Rev. Lett.* **84**, 4031 (2000).
 - [14] E. J. Mueller, T.-L. Ho, M. Ueda, and G. Baym, *Phys. Rev. A* **74**, 033612 (2006).
 - [15] C. V. Ciobanu, S.-K. Yip, and T.-L. Ho, *Phys. Rev. A* **61**, 033607 (2000).
 - [16] M. Koashi and M. Ueda, *Phys. Rev. Lett.* **84**, 1066 (2000); M. Ueda and M. Koashi, *Phys. Rev. A* **65**, 063602 (2002).
 - [17] R. B. Diener and T.-L. Ho, *Phys. Rev. Lett.* **96**, 190405 (2006); L. Santos and T. Pfau, *ibid* **96**, 190404 (2006); H. Mäkelä and K.-A. Suominen, *Phys. Rev. A* **75**, 033610 (2007).
 - [18] Z. F. Xu, Y. Zhang, and L. You, *Phys. Rev. A* **79**, 023613 (2009).
 - [19] Z. F. Xu, J. Zhang, Y. Zhang, and L. You, *Phys. Rev. A* **81**, 033603 (2010).
 - [20] J. Zhang, Z. F. Xu, L. You and Y. Zhang, *Phys. Rev. A* **82**, 013625 (2010).
 - [21] A. Smerzi, S. Fantoni, S. Giovanazzi and S.-R. Shenoy, *Phys. Rev. Lett.* **79**, 4950 (1997).
 - [22] T.-L. Ho and V. B. Shenoy, *Phys. Rev. Lett.* **77**, 2595 (1996).
 - [23] S. Yi, Ö. E. Müstecaplıoğlu, C. P. Sun, and L. You, *Phys. Rev. A* **66**, 011601(R) (2002).